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Hydromagnetic non-Newtonian flow near an oscillating flat plate under the action of a body force

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Abstract : The problem of laminar flow of a viscoelastic incompressible fluid of small electrical conductivity near an infinite insulated flat plate is discussed. The plate oscillating harmonically in its own plane under the action of body force which is varying periodically with time. The system is stressed by a uniform magnetic field normal to the plate and the magnetic lines of force are taken to be fixed relative to the fluid.

The effect of the flow parameters is studied. It is found that both magnetic field and elasticity of the fluid have an important effect on the velocity distribution.

Keywords : Non-Newtonian flow, viscoelastic imcompressible fluid oscillating flat plate.

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1. Introduction

The importance of viscoelastic fluids, such as molten plastics, pulps, and emulsions in everyday chemical engineering practice has motivated many investigators to analyze the behaviour of these fluids in motion. If we have viscoelastic fluid in an external magnetic field, the problem is of greater interest because we then have coupled non-Newtonian and magnetic force effects on the flow field. Magnetic forces can be controlled through changes in the magnetic field. Thus, if a viscoelastic fluid were a conductor of electricity, it would be possible that the magnetic force produced in it could influence the flow in a significant way.

The purpose of the present work is to rediscuss the problem treated by Ong and Nicholls [1], namely, that of the flow of an ordinary Newtonian fluid near an oscillating solid flat wall. The study in [1] neglects the elasticity of the fluid. Since it is now known that all fluids in nature do possess elasticity, no matter how small it may be, for the present investigations, we choose a model for a viscoelastic fluid as given by Rivlin-Ericksen [2].

The first solution for the flow of a purely viscous fluid about an infinite flat wall which executes linear harmonic oscillations parallel to itself was given by Stokes [3] and

Our new format of references has been adopted in this paper.

Rayleigh [4]. Later, Stuart [5] investigated the response of skin friction and temperature of an infinite plate thermometer to fluctuations in the stream with suction at the plate. Rossow [6] extended the work of Stokes and Rayleigh to a viscous incompressible electrically conducting fluid in the presence of an external magnetic field. Choudhary [7] treated a more general case by taking the flat plate to have a suction or injection.

In Section 1, we give the rheological equation of state for a viscoelastic fluid of Rivlin-Ericksen type along with the conservation of mass and momentum equations. Section 2 deals with the reduction of the flow equations fọr the present type of flow situation along with the appropriate boundary conditions. In Section 3 we, use a good assumption to obtain the exact solutions of the problem. The results are discussed in Section 4 and compared with those of Ong and Nicholls for the case of purely viscous liquid and of Stokes for a viscoinelastic liquid and in absence of magnetic field. Some interesting results are reported which bring out the effects of elasticity of the fluid on velocities.

2. Rheological equation of state

Rivlin and Ericksen [2] have shown that if terms of order higher than the second are neglected in the kinematic tensors and their invariants, then the non linear stress-strain relationship takes the form

$$T_{ij} = -P\delta_{ij} + 2\mu E_{ij} - k_o D_{ij} + 4k^* E_i{}^m E_{mj}, \tag{1}$$

where
$$E_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

$$D_{ij} = a_{i,j} + a_{j,i} + 2u_{m,i}u_{,j}{}^m, \tag{2}$$

$$a^i = \frac{\partial u^i}{\partial t} + u^m u^i$$

where the symbols have the usual meaning and a suffix following a comma denotes covariant differentiation with respect to that suffix. The normal summation convention for the repeated suffixes is assumed. It is to be noted that the viscoelastic parameter k_o is positive from thermodynamic consideration. The above eqs. (1) and (2) have to be solved in conjunction with the equation of continuity

$$u_{,i}^i = 0 \tag{3}$$

and the equation of momentum

$$\rho\left[\frac{\partial u^i}{\partial t} + u^m u^i_{,m}\right] = T_j^{ij} + F_i \tag{4}$$

where ρ is the density of the fluid and F_i is the external force acting on the fluid. The positive coefficients μ, k_o, and k^* are known as the viscosity, visco-elasticity and cross-viscosity respectively.

3. Reduction of the flow equations

The magnetic Reynolds number R_e is usually small in case of problems of aeronautical engineering. Under such conditions the induced magnetic field due to the flow may be neglected with respect to the applied magnetic field. It is considered that an infinite insulated flat plate executes linear harmonic oscillations in its own plane in the presence of an externally applied transverse magnetic field of uniform strength H_0 fixed relative to the fluid. The plate is under the imposition of a body force which oscillates with time. As the plate is infinite in length, the physical variables depend only on y, the co-ordinate perpendicular to the wall and t, the time. The pressure P in the fluid is assumed constant.

The equation of continuity (3) subject to the condition $v = 0$ at $y = 0$ leads to the result $v = 0$, everywhere. The boundary layer equation for the hydromagnetic viscoelastic flow under the action of a body force (4) simplifies to

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho} \frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma \mu_e^2 H_0^2}{\rho} u - Ae^{-iwnt}, \tag{5}$$

where u is the velocity parallel to the plate, $\upsilon = \mu/\rho$ the kinematic viscosity, σ the electrical conductivity and μ_e, the magnetic permeability. The term $- \sigma\mu_e^2 H_0^2 u/\rho$ represents the pondermotive force parallel to the plate. The term $- Ae^{iwnt}$ represents the body force, while the term $- (k_0/\rho)(\partial^3 u/\partial y^2 \partial t)$ is due to the non-Newtonian fluid effect.

The boundary conditions are :

$$y = 0, \qquad u = U_0 \cos nt \tag{6}$$

$$y = \infty, \qquad u = 0.$$

Introducing

$$t^* = nt, \qquad \eta = y(n/\upsilon)^{1/2}, \qquad m = \sigma\mu_e^2 H_0^2/\rho, \tag{7}$$

$$m_1 = m/n = R_m^2, \qquad k = k_0 n/\rho\upsilon,$$

we have

$$\frac{\partial u}{\partial t^*} = \frac{\partial^2 u}{\partial \eta^2} - k\frac{\partial^3 u}{\partial \eta^2 \partial t^*} - m_1 u - \frac{A}{n} e^{iwt^*} \tag{8}$$

with the boundary conditions

$$\eta = 0, \qquad u = U_0 \cos t^* \tag{9}$$

$$\eta = \infty, \qquad u = 0$$

4. Solution of the problem

Let the solution of eq. (8) be assumed in the form

$$u(\eta, t^*) = F(\eta)\cos(t^* - s\eta) + Be^{-iwt^*} \tag{10}$$

Substituting in eq. (8), it yields

$$F'' - 2skF' - (s^2 + m_1)F = 0, \tag{11}$$

$$kF'' + 2sF' + (1 - ks^2)F = 0, \tag{12}$$

$$B = -A/\{n(m_1 - iw)\}, \tag{13}$$

where dashes denote differentiation with respect to η. The solution of the differential eq. (11) is

$$F(\eta) = C\exp(\lambda\eta)$$

where $\lambda = [ks - (k^2s^2 + m_1 + s^2)^{1/2}]$ and C is the constant of integration.

Substituting for F (η) into eq. (12), we have

$$4s^4(k^2+1)^2 + 4s^2(k^2+1)(m_1 - k) - (km_1 + 1)^2 = 0.$$

Since s^2 is to remain always positive, hence

$$s^2 = \{[(k - m_1)^2 + (km_1 + 1)^2]^{1/2} - (m_1 - k)\}/\{2(k^2 + 1)\}.$$

Hence

$$s(k, m_1) = \{[(k^2+1)(m_1^2+1)]^{1/2} - (m_1 - k)\}^{1/2}/\{2(k^2+1)\}^{1/2} \tag{14}$$

and the solution is

$$u(\eta, t^*) = C\exp[ks - (k^2s^2 + m_1 + s^2)^{1/2}]\,\eta\cos(t^* - s\eta) - Ae^{-iwt^*}/\{n(m_1 - iw)\} \tag{15}$$

i.e. $$u(\eta, t^*) = C\exp[ks - (k^2s^2 + m_1 + s^2)^{1/2}]\,\eta\cos(t^* - s\eta) - A(m_1\cos wt^* + w\sin wt^*)/\{n(m_1^2 + w^2)\} \tag{16}$$

the imaginary part is being neglected for obvious reasons.

The first of the boundary conditions (9) gives

$$C = U_o + A(m_1\cos wt^* + w\sin wt^*)/\{n(m_1^2 + w^2)\cos t^*\}. \tag{17}$$

Substituting from (17) in (16), the velocity distribution is seen to be given by the following equation

$$u(\eta, t^*) = [U_o + A(m_1\cos wt^* + w\sin wt^*)/\{n(m_1^2+w^2)\cos t^*\}$$
$$\exp[ks - (k^2s^2 + m_1 + s^2)^{1/2}]\,\eta\cos(t^* - s\eta)$$
$$- A(m_1\cos wt^* + w\sin\cdot wt^*)/\{n(m_1^2 + w^2)\} \tag{18}$$

where s is defined by (14).

4. Discussion of the results

The velocity distribution for the flow of a viscoelastic incompressible fluid of small electrical conductivity near an infinite oscillating flat plate in the presence of a transverse magnetic field fixed relative to the fluid and under the imposition of a body force which varies periodically with time, is given by eq. (18).

If we put $k = 0$, we get the results for hydromagnetic Newtonian flow near an oscillating solid flat wall under the imposition of a body force varying periodically with time as by Ong and Nicholls [1].

If we take $k = 0$ and $m_1 = 0$, we get the results for the flow of a Newtonian viscous, incompressible non-conducting fluid near an infinite oscillating solid flat plate under the imposition of a body force varying periodically with time as studied by Stokes [3].

If $m_1 = 0$ and $k \neq 0$ (this problem was not treated before) eq. (18) becomes

$$u(\eta, t^*) = U_0 + A \sin wt^*/nw \cos t^*)x \,.$$
$$\exp\{-\eta \{[k + (k^2+1)^{1/2}]/[2(k^2+1)]\}^{1/2},$$
$$\cos\{t^* - \eta\{[(k^2+1)^{1/2} + k]/[2(k^2+1)]\}^{1/2}\} - A \sin wt^*/nw,$$

which is the solution for the flow of a non-conduction non-Newtonian viscoelastic fluid near an oscillating solid flat plate under the imposition of a body force which varies periodically with time.

For numerical computation, we have taken

$$t^* = 0, \pi/4, U_0 = 2, A = 2, n = 1, w = 1, m = 0, 2 \text{ and } k = 0 \text{ and } 1.8.$$

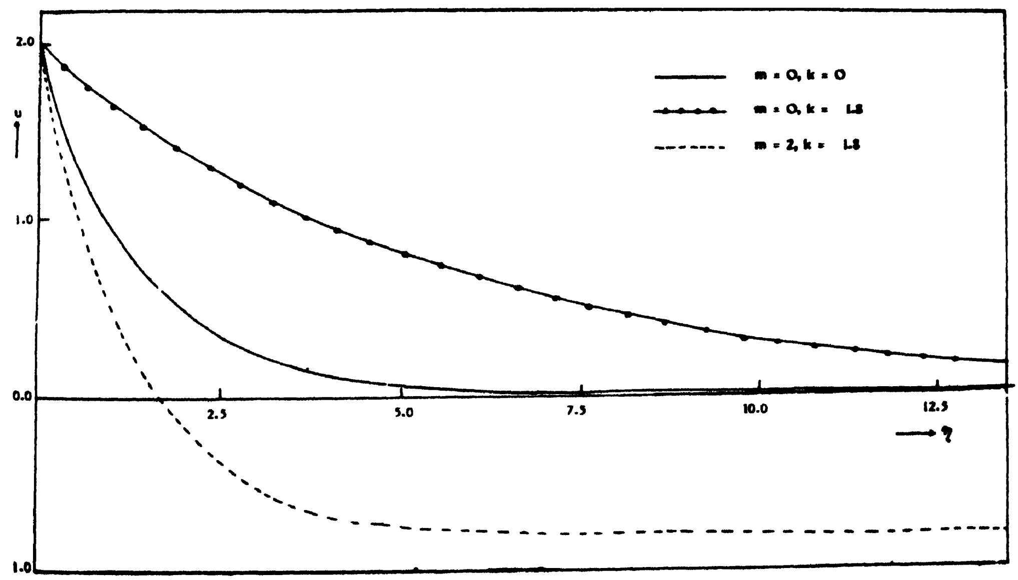

Figure 1. Velocity distribution at $t^* = 0$ for various values of m and k.

The behaviour of the velocity profile with the variation of various parameters are shown graphically. As seen in Figures 1 and 2 the non-Newtonian property of viscoelastic fluid works to increase the velocity profile, while the magnetic effect is to decrease the velocity distribution.

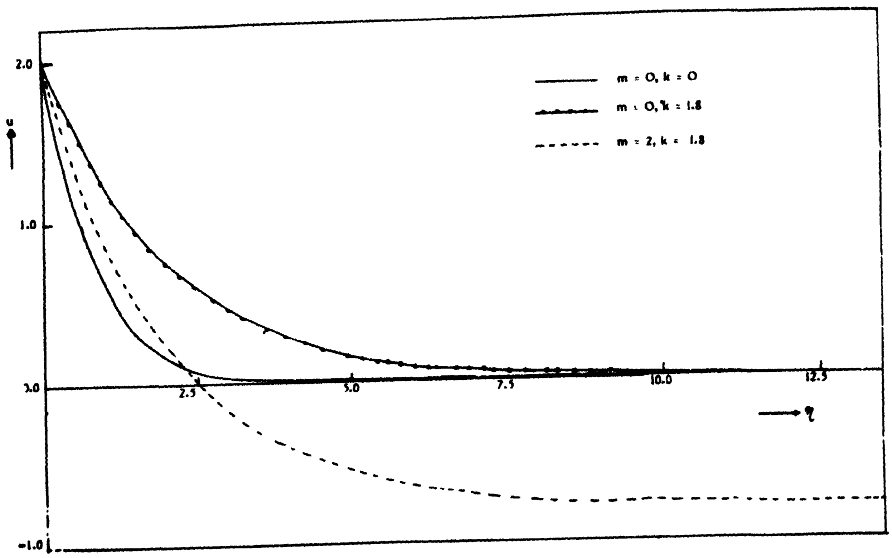

Figure 2. Velocity distribution at $t^* = \pi/4$ for various values of m and k.

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